Practice Problems for the Final Exam

Q1. Determine whether each of the following series are convergent or divergent. You may use any applicable test covered in this course.

(1) $\sum_{n=0}^{\infty} \arctan(n^2)$

(2) $\sum_{n=1}^{\infty} \frac{1}{n^2+n+1}$

(3) $\sum_{n=0}^{\infty} \frac{(n+1)(3^2-1)^n}{3^{2n}}$

(4) $\sum_{n=0}^{\infty} \frac{2n^2+3n+4}{\sqrt{n^5+6n^2+7}}$

(5) $\sum_{n=2}^{\infty} \frac{\ln(n)}{n}$

(6) $\sum_{n=1}^{\infty} \ln\left(\frac{2n^2+3}{n^2+n}\right)$

(7) $\sum_{n=0}^{\infty} \frac{(-3)^n}{2^{2n+1}}$

(8) $\sum_{n=2}^{\infty} \left(\frac{1}{n}-\frac{1}{n+1}\right)$

(9) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{(n+1)^n}$

(10) $\sum_{n=0}^{\infty} \frac{(-9)^n}{6^{n+1}}$

(11) $\sum_{n=0}^{\infty} \frac{2^n}{4^n+1}$

(12) $\sum_{n=1}^{\infty} \frac{3^{n-1}}{n(2^n+1)}$

(13) $\sum_{n=1}^{\infty} \frac{n^2}{n!}$

(14) $\sum_{n=5}^{\infty} \frac{2n^2-1}{n^4-n^3+n^2+1}$

(15) $\sum_{n=0}^{\infty} \frac{1}{n!}$

(16) $\sum_{n=1}^{\infty} \frac{1}{n^{\sin(1)}}$

(17) $\sum_{n=2}^{\infty} \frac{(-1)^n}{(\ln(n))^2}$

(18) $\sum_{n=0}^{\infty} \frac{4^{n-2}(3^n)}{8^{2n+1}+2}$

(19) $\sum_{n=1}^{\infty} \frac{\cos(\pi n)}{n^2}$

(20) $\sum_{n=2}^{\infty} \frac{\ln(n)}{n^2}$

(21) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{\sqrt{n}}$

(22) $\sum_{n=1}^{\infty} \frac{n^2+n+1}{n+2}$

(23) $\sum_{n=1}^{\infty} (-1)^n \sin\left(\frac{1}{n}\right)$

(24) $\sum_{n=0}^{\infty} \frac{(-1)^n}{(n+1)!}$

(25) $\sum_{n=3}^{\infty} \big(\ln(n)-\ln(n+1)\big)$

(26) $\sum_{n=0}^{\infty} \frac{n!}{3^{n+1}}$

(27) $\sum_{n=1}^{\infty} \frac{\cos^2(n)\sqrt{n}}{n^2}$

(28) $\sum_{n=1}^{\infty} (-1)^{n-1}\frac{\sqrt{n}}{n+1}$

(29) $\sum_{n=1}^{\infty} \frac{(5n^2+5)(-9)^n}{10^{n+3}}$

(30) $\sum_{n=1}^{\infty} \left(\frac{n+9}{5n+6}\right)^n$

(31) $\sum_{n=1}^{\infty} \frac{e^n}{n!}$

(32) $\sum_{n=2}^{\infty} \frac{1}{n\ln(n)}$

(33) $\sum_{n=2}^{\infty} \frac{n^3+n^2+n+1}{2n^2-3n+4}$

(34) $\sum_{n=2}^{\infty} \cos\left(\frac{\pi}{n}\right)$

(35) $\sum_{n=2}^{\infty} \frac{(\ln(n))^4}{n+3}$

(36) $\sum_{n=1}^{\infty} \left(\frac{2n^2+6n}{8n^4+4}\right)^3$

Q2. Give approximations of the following series up to the specified accuracy. Your answer must be justified by some estimation theorem covered in class.

(1) $\sum_{n=0}^{\infty} \frac{(-1)^n}{n^2+1}$ with error ≤ 0.005.

(2) $\sum_{n=1}^{\infty} \frac{1}{n^3}$ up to 3 decimal places.

(3) $\sum_{n=1}^{\infty} \frac{(-3)^{2n+1}}{(2n+1)!}$ up to 3 decimal places.

(4) $\sum_{n=1}^{\infty} ne^{-n}$ with error $\leq 10^{-3} = 0.001$.

Q3. Find a power series representation for the following functions and identify an open interval of convergence (i.e. don't bother with the endpoints, i.e. giving the radius of convergence R suffices).

(1) $f(x) = \arctan(x)$

(2) $g(x) = \frac{1}{(1-x)^2}$

(3) $h(x) = \frac{x^2}{1+x^2}$

(4) $a(x) = x^2\sin(x)$

(5) $b(x) = \int_0^2 \cos(x^2)\,dx$

(6) $c(x) = \frac{d}{dx}\left(\frac{x^2}{1-x^3}\right)$

Q4. Determine the radius of convergence R and the interval of convergence I for the following power series.

(1) $\displaystyle\sum_{n=0}^{\infty}(3x-2)^n$ **(2)** $\displaystyle\sum_{n=1}^{\infty}\frac{x^n}{2n+1}$ **(3)** $\displaystyle\sum_{n=1}^{\infty}\frac{x^{2n+1}}{(n+1)!(2n+1)}$

Q5. Approximate the following integrals up to the specified accuracy. Your answer must be justified by some estimation theorem covered in class.

(1) $A_1 = \displaystyle\int_0^1 x^2 e^{-x^2}\,dx$ up to 3 decimal places.

(2) $A_2 = \displaystyle\int_0^1 \sin(x^2)\,dx$ up to 3 decimal places.

(3) $A_3 = \displaystyle\int_0^2 \cos(x^2)\,dx$ with error ≤ 0.001.

(4) $A_4 = \displaystyle\int_0^{\frac{3}{4}} \arctan(x^2)\,dx$ with error ≤ 0.0001.

(5) $A_5 = \displaystyle\int_0^{\frac{2}{3}} x\ln(1+x^3)\,dx$ up to 5 decimal places.

(6) $A_6 = \displaystyle\int_0^1 e^{-x^2}\,dx$ with error ≤ 0.0005.